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ND Tadpoles as New String States and Quantum Mechanical Particle-Wave Duality from World-Sheet T-Duality

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Abstract. We consider new objects in bosonic open string theory – ND tadpoles, which have N(euman) boundary conditions at one end of the world-sheet and D(irichlet) at the other, must exist due to $s - t$ duality in a string theory with both NN strings and D-branes. We demonstrate how to interpolate between N and D boundary conditions. In the case of mixed boundary conditions the action for a quantum particle is induced on the boundary. Quantum-mechanical particle-wave duality, a dual description of a quantum particle in either the coordinate or the momentum representation, is induced by world-sheet T-duality. The famous $\tilde{R} = \alpha/R$ relation is equivalent to the quantization of the phase space area of a Planck cell $\oint pdx = 2\pi\hbar$. We also introduce a boundary operator \mathcal{Z} - a “Zipper” which changes the boundary condition from N into D and vice versa.

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Recently the theory of open strings, which was overshadowed for many years by the explosion of interest in closed strings, has attracted renewed interest largely because of the discovery of D-branes [1, 2]. Let $\sigma \in [0, \pi]$ be the normal coordinate and τ be the tangential coordinate on an open string world-sheet (fig.1) and define $z = e^{\tau+i\sigma}$. A normal open string configuration $X^\mu(z, \bar{z})$ has Neuman boundary conditions, $\partial_\sigma X^\mu = 0$ at $\sigma = 0, \pi$. On the other hand a D-brane has Dirichlet boundary conditions $\partial_\tau X^\mu = 0$ at $\sigma = 0, \pi$. However, because an open string has two boundaries, one at $\sigma = 0$ and one at $\sigma = \pi$ it is also possible to have Dirichlet boundary conditions at one boundary and Neuman at the other. The respective mode expansions are given by (for a review see [2] and references therein)

$$\begin{aligned}
 \text{NN} \quad X^\mu(z, \bar{z}) &= x^\mu - i'\alpha' p^\mu \ln z\bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m} (z^{-m} + \bar{z}^{-m}) \\
 \text{DD} \quad X^\mu(z, \bar{z}) &= -i\frac{\delta X^\mu}{2\pi} \ln \frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m} (z^{-m} - \bar{z}^{-m}) \\
 \text{ND, DN} \quad X^\mu(z, \bar{z}) &= i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} (z^{-m-\frac{1}{2}} \pm \bar{z}^{-m-\frac{1}{2}})
 \end{aligned} \tag{1}$$

although, as we will argue later, the expression for ND is incomplete. NN corresponds to open strings with free ends (which move at the speed of light). DD strings are open strings attached to a fixed surface or point – the D-brane (in this case δX^μ is the displacement between two endpoints of an open string). ND strings, we will call them tadpoles, correspond to an open string which is attached at one end to a D-brane while the other end is free.

ND boundary conditions have been studied earlier, both in string theory [3, 4, 5] and in $c = 1$ boundary conformal field theory [6]. Here we take the new step of stressing that the ND tadpoles must be considered on an equal physical footing with the usual NN open strings and the D-branes. By analogy with what occurs in ordinary field theory it is common to think of the NN strings as the fundamental quanta, f , and of the D-branes as the solitons, S , of the theory; however we then expect the process

$$f\bar{f} \rightarrow S\bar{S} \tag{2}$$

to occur. There are two possible ways to describe this process in string theory. The

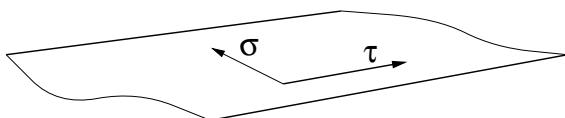


Figure 1: The world sheet coordinates σ and τ .

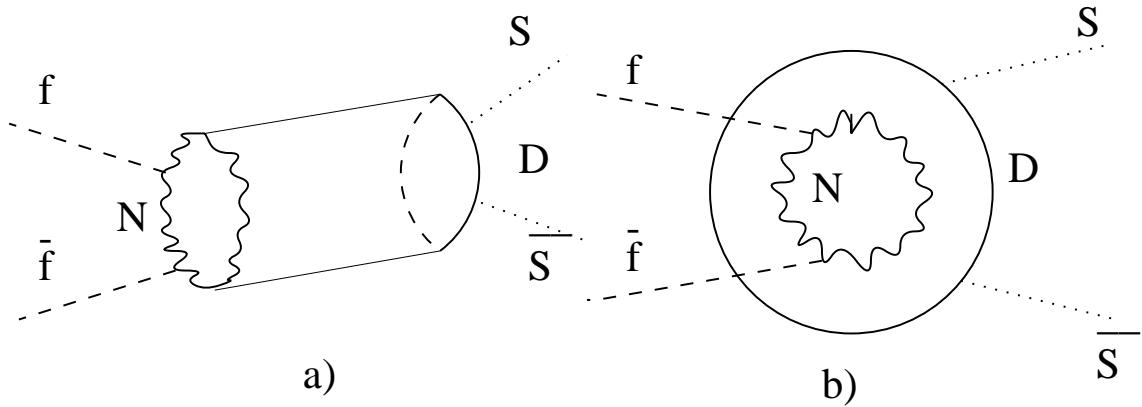


Figure 2: a) Soliton pair production from fundamental quanta through a closed string intermediate state and b) the corresponding t -channel process contains an ND tadpole in the intermediate state.

first one is through the closed string world-sheet interpolating between N and D closed boundaries (Fig.2a). One can look at this either as a transition between N and D boundaries via the closed string or as a loop of an ND tadpole (Fig.2b). In this case the quantum numbers of the initial $f\bar{f}$ pair (as well as the final $S\bar{S}$ pair) must be compatible with the quantum numbers of closed string states (graviton, dilaton, etc). When this is not true, for example if instead of e^+e^- annihilation into monopole-antimonopole pair one has $\bar{\nu}e^-$ annihilation into monopole-antidyon pair, it is impossible to connect initial and final states by a closed string state. In this case the world sheet for this process must look like Fig.3a. We have introduced an operator \mathcal{Z} which we will call the “Zipper” which acts on the boundary to change N boundary conditions into D (there is correspondingly an anti-zipper which changes D to N). In the t -channel the diagram becomes Fig. 3b which is nothing but the propagation of an ND tadpole in the intermediate state. We conclude that in a theory able to describe the transition between fundamental quanta and solitons the ND tadpoles must be present because of standard $s-t$ crossing symmetry. ND states also appear as a consequence of the modular invariance of NN and DD systems as was first observed in [4] where the one-loop partition function was studied on open string orbifolds. Note that even in the first case of Fig.2a the Zipper is a natural object. The problem is that if it is necessary to produce two D -branes at two different points in space (for example in pair production other than at threshold) it is very hard to imagine a boundary condition along the D boundary where X takes one value X_1 on one part of the boundary and then suddenly is changed into X_2 along another part. This can be done easily by inserting a Zipper - un-Zipper pair $\mathcal{Z} - \bar{\mathcal{Z}}$ which will transform D conditions with boundary value X_1 into N conditions and then back into D conditions but with a new value X_2 .

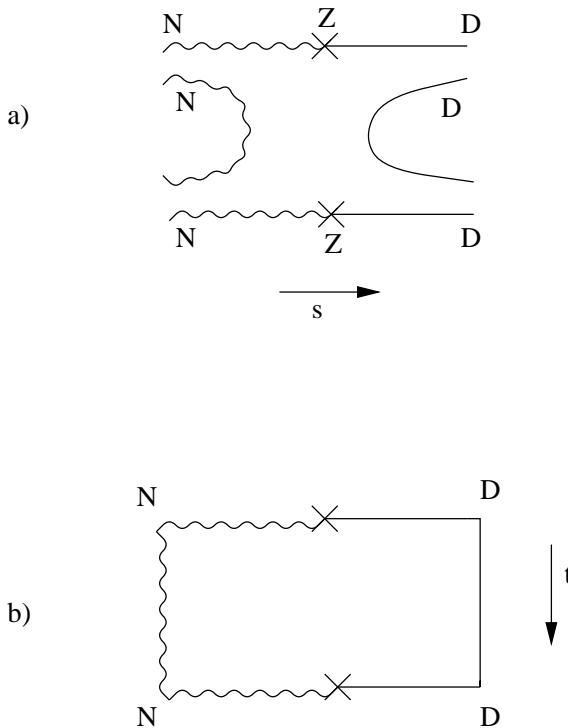


Figure 3: Production of $S\bar{S}$ through an open string intermediate state in a) the s-channel and b) the t-channel. Crosses mark the Zipper.

Strings originated in the study of hadronic physics; more recently models of hadrons motivated by QCD in which quarks are bound together by tubes of non-abelian flux have been developed (see [7] and references therein). Using the language of quarks and hadrons one can say that NN strings are $q\bar{q}$ meson states with massless quarks, DD are $Q\bar{Q}$ quarkonia systems with infinitely heavy quarks and ND tadpoles are the $Q\bar{q}$ mesons – the “B” mesons, or heavy-light mesons, of QCD. Interestingly even baryons in QCD can be described in the same spirit as a combination of three ND tadpoles with a common D boundary which is the centre of mass; once again the baryon is a sort of soliton (see fig.4b which actually shows a SU(4) baryon for technical reasons). The center of mass, sometimes called the string junction, is a soliton collective coordinate. The dynamics of this object was discussed in [8]. The Zipper operator is the key to describing weak decays of mesons; its ability to turn D into N parallels the quark process $Q\bar{Q} \rightarrow Q\bar{q} + W$ (fig.4a); in the stringy language the W vertex must contain \mathcal{Z} .

The existence of \mathcal{Z} which changes N to D raises the question of the interpolation between N and D. A more general boundary condition is

$$f(\tau)\partial_\sigma X^\mu + X^\mu = \text{const} \quad (3)$$

If $f = \infty$ we get N and if $f = 0$ we get D boundary conditions; however the more

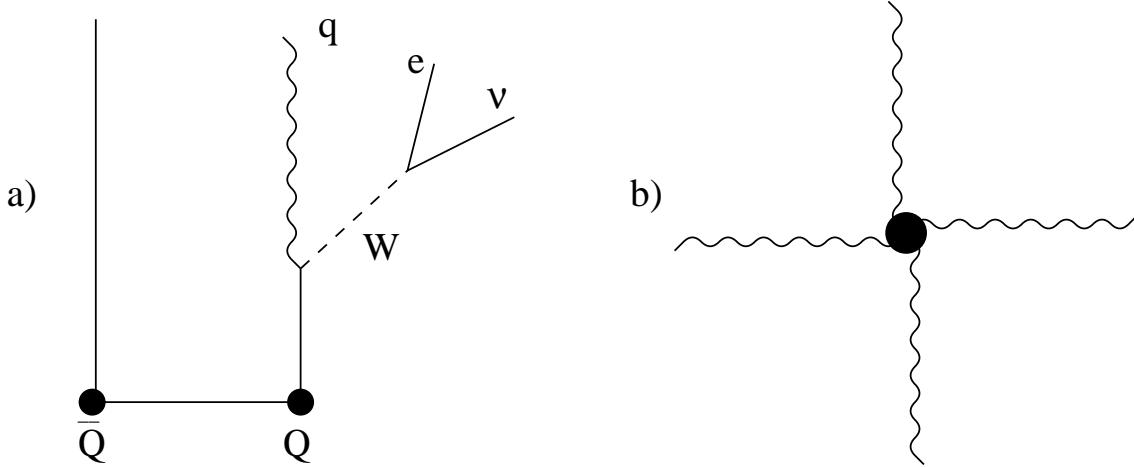


Figure 4: a) Semi-leptonic weak decay of heavy quarkonia and b) An $N_c = 4$ baryon.

general condition (3) is not consistent with the unmodified string action which is, in the conformal gauge,

$$\mathcal{S} = \frac{1}{2} \int_{\Sigma} (\partial_a X^{\mu} \partial_a X^{\mu}) d^2 \xi \quad (4)$$

The variation of \mathcal{S} under $X \rightarrow X + \delta X$ is

$$\delta \mathcal{S} = - \int_{\Sigma} \delta X^{\mu} \partial^2 X^{\mu} d^2 \xi + \int_{\partial \Sigma} \delta X^{\mu} \partial_{\sigma} X^{\mu} d\tau \quad (5)$$

and the condition for an extremum is that either $\delta X^{\mu} = 0$ or $\partial_{\sigma} X^{\mu} = 0$ on the boundary. To get (3) we must add an extra term to the action and consider

$$\mathcal{S}' = \frac{1}{2} \int_{\Sigma} (\partial_a X^{\mu} \partial_a X^{\mu}) d^2 \xi + \frac{1}{2} \int_{\partial \Sigma} f(\tau) (\partial_{\sigma} X^{\mu})^2 d\tau \quad (6)$$

whose variation is

$$\delta \mathcal{S}' = - \int_{\Sigma} \delta X^{\mu} \partial^2 X^{\mu} d^2 \xi + \int_{\partial \Sigma} (\delta X^{\mu} + f(\tau) \delta (\partial_{\sigma} X^{\mu})) \partial_{\sigma} X^{\mu} d\tau \quad (7)$$

Now we have an extremum when X^{μ} satisfies Laplace's equation and the boundary condition

$$\delta (X^{\mu} + f(\tau) \partial_{\sigma} X^{\mu}) = 0 \quad (8)$$

which is the same as (3).

To understand the meaning of this extra term in the action we shall make a T-duality transformation; this maps X to a new field Φ with the relation

$$\partial_i X^{\mu} = \epsilon_{ij} \partial_j \Phi^{\mu} \quad (9)$$

With this change of variables \mathcal{S}' becomes

$$\tilde{\mathcal{S}}' = \frac{1}{2} \int_{\tilde{\Sigma}} (\partial_a \Phi^{\mu})^2 d^2 \xi + \frac{1}{2} \int_{\partial \tilde{\Sigma}} f(\tau) (\partial_{\tau} \Phi^{\mu})^2 d\tau \quad (10)$$

Now the boundary term resembles the action for the path integral describing the motion of a free particle along the trajectory $\Phi^\mu(\tau)$. By considering the variation $\Phi \rightarrow \Phi + \delta\Phi$ we find that Φ satisfies Laplace's equation but this time with the boundary conditions

$$\partial_\sigma \Phi^\mu - \partial_\tau (f(\tau) \partial_\tau \Phi^\mu) = 0 \quad (11)$$

Using (9) we can write this in terms of X as

$$\partial_\tau (X^\mu + f(\tau) \partial_\sigma X^\mu) = 0 \quad (12)$$

which is the same as (3). If $f = \infty$ we get D and if $f = 0$ we get N boundary conditions for Φ^μ .

We see that by introducing mixed boundary conditions we have induced some dynamics on the world-line formed by the boundary of the world-sheet. This is the same phenomenon as in the case of a three-dimensional topologically massive gauge theory which induces (depending on boundary conditions) a non-trivial dynamics on a three-dimensional boundary (see for example [9] and references therein). It is very tempting to describe this induced boundary dynamics as a massive relativistic particle with mass proportional to f . Let us demonstrate that this is precisely the case.

Consider the Feynman path integral representation for the relativistic propagator of a scalar particle of mass M in d space-time dimensions (for simplicity consider a Wick rotated propagator in the Euclidean region)

$$\begin{aligned} G(x, y) &= \int \frac{dp}{(2\pi)^d} \frac{e^{ip(x-y)}}{p^2 + M^2} = \frac{1}{2} \int \frac{dp}{(2\pi)^d} e^{ip(x-y)} \int_0^\infty dT e^{-(p^2 + M^2)T/2} = \\ &= \frac{1}{2} \int_0^\infty dT e^{-M^2 T/2} \int_{x(0)=x}^{x(T)=y} \mathcal{D}x(\tau) \exp\left[-\frac{1}{2} \int_0^T d\tau \dot{x}^2\right] \end{aligned} \quad (13)$$

It is possible to represent the integration over the proper time T as an integration over one-dimensional metrics $h(\tau)$ modulo the one-dimensional reparametrization group $f(\tau)$ (see, for example, [10]), so the relativistic scalar propagator takes the form

$$\begin{aligned} G(x, y) &= \int \mathcal{D}x(\tau) \mathcal{D}h(\tau) \exp\left[-\frac{1}{2} \int d\tau \frac{\dot{x}^\mu \dot{x}_\mu}{\sqrt{h(\tau)}} - \frac{1}{2} M^2 \int d\tau \sqrt{h(\tau)}\right] = \\ &\int \mathcal{D}x(\tau) \mathcal{D}p(\tau) \mathcal{D}h(\tau) \exp\left[i \int d\tau p^\mu(\tau) \dot{x}_\mu(\tau) - \frac{1}{2} \int d\tau \sqrt{h(\tau)} (p^\mu(\tau) p_\mu(\tau) + M^2)\right] \end{aligned} \quad (14)$$

where the coordinate along the world-line is τ and $h(\tau)$ is the one-dimensional metric. The proper time is the only reparametrization invariant characteristic of the metric

– the length of the path $\int_0^1 \sqrt{h} d\tau = T$. Let us now change the variable τ into a new time $t(\tau)$ in such a way that

$$\frac{dt}{d\tau} = M^2 \sqrt{h(\tau)} \quad (15)$$

after which the action along the trajectory $x^\mu(\tau)$ in the first equation of (14) can be rewritten as

$$\frac{1}{2} M^2 \int dt \frac{dx^\mu}{dt} \frac{dx_\mu}{dt} - \frac{1}{2} \int dt \quad (16)$$

where the second term is some constant and can be dropped. Comparing this with (10) one can see that Φ^μ describes the target-space coordinate of a massive particle and that f is indeed related to M (actually M^2) by

$$f(\tau) \frac{dt}{d\tau} = M^2 \quad (17)$$

But if Φ^μ describes the target-space coordinate of a massive particle, what is the role of its dual field X^μ ?

To answer this question we have to rewrite the boundary action (6) not in terms of $\partial_\sigma X$ but in terms of X . Fortunately we can do this using the boundary condition (3) $X = -f \partial_\sigma X$ where X in (3) is shifted to put $const = 0$. Then the action can be written as

$$\frac{1}{2} \int_{\partial\Sigma} \frac{1}{f(\tau)} X^\mu(\tau) X_\mu(\tau) d\tau = \frac{1}{2M^2} \int_{\partial\Sigma} X^\mu(t) X_\mu(\tau) dt \quad (18)$$

where we used (17) again. Comparing this with the second line in (14) and using (15) we immediately see that $X^\mu(\tau)$ is the momentum $p(\tau)$.

This is a very remarkable fact – we just demonstrated that the coordinates X^μ and Φ^μ are the coordinates in *phase space* and that the T-duality transformation on the world-sheet induces a transformation in a target- phase space! In other words T-duality is related to de Broglie quantum-mechanical “particle - wave” duality.

Note that by definition T-duality relates “coordinates” for X and “momenta” for Φ and vice versa because of the relation (9) – indeed the spatial derivative (“coordinate”) of one of the fields is related to the time derivative (“conjugate momentum”) of the other one. However it is important to stress that what we have just found is much more amusing – we obtained a quantum description of physics in target space, even in the limit when the string coupling is zero. Naively we must have classical physics in target space; nevertheless the world-sheet already knows about quantum reality.

The relationship between compactification radii for X and Φ , $R \rightarrow \tilde{R} = \alpha'/R$, implies the famous quantization condition of a Planck cell in phase space:

$$\oint pdx = 2\pi\hbar \quad (19)$$

This arises because X and Φ (or, more precisely, their zero modes) play the roles of phase space coordinates p and x ; recalling that there is a $1/2\pi\alpha'$ factor in front of the world-sheet action we have

$$x = \Phi; \quad p = \frac{\hbar}{2\pi\alpha'} X \quad (20)$$

Taking into account that $\Phi \in (0, 2\pi R)$ and $X \in (0, 2\pi\tilde{R})$ we can easily see that

$$\begin{aligned} \oint pdx &= \frac{\hbar}{2\pi\alpha'} (2\pi R)(2\pi\tilde{R}) = (2\pi\hbar) \frac{R\tilde{R}}{\alpha'} \\ \oint pdx &= (2\pi\hbar) \Rightarrow \tilde{R} = \frac{\alpha'}{R} \end{aligned} \quad (21)$$

Now we can write down an expression for the Zipper operator – it is nothing but the string generalization of the $p\dot{x}$ term in the path integral (14). One can define it as (we introduce explicitly the factor $1/2\pi\alpha'$)

$$\mathcal{Z} = \int \mathcal{D}X(\tau) \mathcal{D}\Phi(\tau) \exp \left[\frac{i}{2\pi\alpha'} \int d^2\xi \epsilon_{ab} \partial_a X \partial_b \Phi \right] \quad (22)$$

and this is precisely what changes N into D – but as we have already seen this is nothing but a change of the quantum-mechanical description from the x - into the p - representation. The density $\epsilon_{ab} \partial_a X \partial_b \Phi$ is a total derivative so the exponential factor is reduced to an integral along the boundary

$$\frac{1}{2\pi\alpha'} \int d\tau X(\tau) \dot{\Phi}(\tau) \quad (23)$$

which corresponds to $\int d\tau p\dot{x}$ term. To change boundary conditions from N to D at some point z on the boundary we have to provide the boundary conditions for fields X and Φ such that $X = 0$ for all $\tau < z$ and $\Phi = 0$ for all $\tau > z$; for the un-Zipper $\bar{\mathcal{Z}}$ one has to exchange boundary conditions for X and Φ . Let us note that due to the condition (21) the world-sheet action in the definition of the Zipper operator does not depend on the homotopy class of the maps $X(\xi)$ and $\Phi(\xi)$ – if one has a map where both X and Φ wrap around the respective circles with circumferences $2\pi R$ and $2\pi\tilde{R}$, there will be an extra factor

$$\exp \left[2\pi i \frac{\tilde{R}R}{\alpha'} \right] = 1 \quad (24)$$

which means that the operator (22) is well-defined.

One can use these operators to define vertex operators at world-sheet boundaries with D boundary conditions. Usually it is impossible to transfer any momentum

at the D boundary because the standard exponential operator $\exp(ik_\mu X^\mu)$ does not exist as a fluctuating field along the boundary. The only way to transfer momentum to the D-brane was through the bulk vertex operators, i.e. we could scatter closed strings, but not open ones, off the D-brane. Now we can do something new, namely insert a pair $\mathcal{Z} - \bar{\mathcal{Z}}$ at the D boundary and create a short interval with N boundary conditions, where we can insert any vertex operator $V_N(\tau)$. Shrinking the size of the N interval to zero we can define new vertex operators which can then be integrated along the D boundary

$$\int d\tau V_D(\tau) = \lim_{\epsilon \rightarrow 0} \int d\tau \bar{\mathcal{Z}}(\tau + \epsilon) V_N(\tau) \mathcal{Z}(\tau - \epsilon) \quad (25)$$

These new vertex operators will enable us to study the dynamics of D-branes.

A further indication of non-trivial connections between target space and world-sheet induced by T-duality comes from considering the ND tadpoles. In the case of NN and DD strings one can either study T-duality, in which case NN and DD strings are interchanged, or a world-sheet parity transformation $\sigma \rightarrow \pi - \sigma$ which exchanges the boundaries and leaves both types of strings intact. It seems that T-duality and world-sheet parity transformation are unrelated, but this is wrong. It is quite clear that both T-duality and a world-sheet parity transformation transform the ND tadpole into the DN tadpole and vice versa. Let

$$\begin{aligned} X_+^\mu(z) &= x_+^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} \left(z^{-m-\frac{1}{2}} \right) \\ X_-^\mu(\bar{z}) &= x_-^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} \left(\bar{z}^{-m-\frac{1}{2}} \right) \end{aligned} \quad (26)$$

Then a tadpole with N boundary conditions at $\sigma = 0$ and D boundary conditions at $\sigma = \pi$ is

$$\begin{aligned} X^\mu(z, \bar{z}) &= X_+^\mu(z) + X_-^\mu(\bar{z}) \\ &= x_+^\mu + x_-^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} \left(z^{-m-\frac{1}{2}} + \bar{z}^{-m-\frac{1}{2}} \right) \end{aligned} \quad (27)$$

A world-sheet parity transformation $\sigma \rightarrow \pi - \sigma$ transforms this to a tadpole with D boundary conditions at $\sigma = 0$ and N boundary conditions at $\sigma = \pi$

$$\mathcal{P}X^\mu(z, \bar{z}) = x_+^\mu + x_-^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} (-1)^m i \left(\bar{z}^{-m-\frac{1}{2}} - z^{-m-\frac{1}{2}} \right) \quad (28)$$

However a T transformation also has the same effect giving

$$\begin{aligned} \Phi^\mu(z, \bar{z}) &= X_+^\mu(\bar{z}) - X_-^\mu(z) \\ &= x_+^\mu - x_-^\mu + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\tilde{\alpha}_m^\mu}{m} \left(\bar{z}^{-m-\frac{1}{2}} - z^{-m-\frac{1}{2}} \right) \end{aligned} \quad (29)$$

where the dual oscillators $\tilde{\alpha}_m = i(-1)^m \alpha_m$. Note that this ND tadpole is in a different position in target space ($x_+^\mu - x_-^\mu$ rather than $x_+^\mu + x_-^\mu$); the commutator of a T transformation and a world sheet parity \mathcal{P} operation is a target space translation. Thus we have to add to the ND mode expansion in (1) a zero mode term which reflects the fact that the center of mass of the ND tadpole can be anywhere in target space.

From another point of view the mixed boundary conditions (3) generate a world sheet boundary state with mass f^{-1} ; this is easily seen by considering the mode expansion which yields a solution

$$X^\mu(z, \bar{z}) = x_+^\mu + x_-^\mu + ip^\mu \log \frac{z}{\bar{z}} + i\sqrt{\frac{\alpha'}{2}} \sum_m \frac{\alpha_m^\mu}{m} \left(z^{-m} + \frac{1 - ifm}{1 + ifm} \bar{z}^{-m} \right) \quad (30)$$

As usual the dual field is obtained by considering, instead of the sum of left and right movers $X_+^\mu(z) + X_-^\mu(\bar{z})$, the difference $X_+^\mu(\bar{z}) - X_-^\mu(z)$ with the simultaneous exchange of z and \bar{z} . The appearance of a pole at $m = if^{-1}$ in (30) means that the world sheet two point function $\langle XX \rangle$ gains a contribution decaying exponentially with (world sheet) distance from the boundary; there is a massive state living on the boundary. It is interesting to note that this mass is, up to a factor of α' , the inverse of the target space mass we found above; this suggests that the world sheet effect should be regarded as resulting from the gravitation induced by the target space mass, i.e. comparing world-sheet and target-space scales (in light-cone gauge, for example), then this scale is not a Compton length $\lambda_C = \hbar/M$, but a gravitational (Schwarzschild) radius $\lambda_G \sim \alpha' M$. The dynamics of this boundary state is very interesting and we hope to discuss it further in future publications. Recently the boundary states in one-dimensional Hubbard and t-J models were considered in [11] where their existence was related to the mixed boundary conditions.

There are several interesting problems we hope to address in the future. The first one is to include world-sheet gravity. In this case the one-dimensional boundary action will be equipped with a naturally induced metric $h(\tau)$ and the integral over this metric will arise as a part of a path integral over the 2d metric $g_{ab}(\xi)$ on a world-sheet with a boundary. It is an interesting problem to study the interaction of the boundary state with the 2d and induced 1d gravity and, for example, to see how the mixed boundary conditions will affect the conformal anomaly. It is quite possible that there will be boundary states in the gravitational sector too. Another problem is to include supersymmetry and world-sheet fermions and to demonstrate that the action for a supersymmetric quantum-mechanical particle will be induced at the boundary. It is also interesting to look at mixed boundary conditions and induced boundary dynamics in rigid strings [10].

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